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# THEORY OF STATIONARY PROPERTIES OF A FULLY-IONIZED CIRCUMTERRESTRIAL PLASMA

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# t: THEORY OF STATIONARY PROPERTIES OF A FULLY IONIZED CIRCUMTERRESTRIAL PLASMA [ Russian title ]*

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Are the circumterrestrial radiation belts of a unitary statistical formation structure? To answer this question it is important to account for the following:

1) the ellipsoidicity of the distribution functions at great heights;

2) the reactions of this distribution upon the action of the outer (dipole) magnetic field of the Earth;

3) interaction of charged particles among themselves and with the aggregate charge of the Earth and of the atmosphere.

The requirement of distribution functions' elipsoidicity is dictated by the denial of atmosphere rotation at great heights with a simultaneous preservation of temperature distribution. The indicated factors lead to a strong spatial anisotropy in the distribution of electrons and nuclei, which can be precalculated and compared with experimentally observable data on Earth's radiation belts.

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* [ • TEORIYA STATSIONARNYKH SVOSTV POLNOST'YU IONIZIROVANNO OKOLOZEMNOY PLAZMY. ]

The fundamental characteristics of the distribution of particles are determined by the constant ellipsoidicity A. The knowledge of temperatures and concentrations of particles at a single point is sufficient for its numerical computation. The quantitative problem setup and the method for its solution are quite analogus [1].

The distribution functions of nuclei and electrons are described by the system of equations:

$$\frac{\partial f_\pm}{\partial t} + v_r \frac{\partial f_\pm}{\partial r} + \frac{v_\vartheta}{r}\frac{\partial f_\pm}{\partial \vartheta} + \frac{v_\varphi}{r\sin\vartheta}\frac{\partial f_\pm}{\partial \varphi} + \left(\frac{v_\vartheta^2 + v_\varphi^2}{r}\right)\frac{\partial f_\pm}{\partial v_r} +$$
$$+ \left(-\frac{v_r v_\vartheta}{r} + \operatorname{ctg}\vartheta\,\frac{v_\varphi^2}{r}\right)\frac{\partial f_\pm}{\partial v_\vartheta} + \left(-\frac{v_r v_\varphi}{r} - \operatorname{ctg}\vartheta\,\frac{v_\vartheta v_\varphi}{r}\right)\frac{\partial f_\pm}{\partial v_\varphi} -$$
$$- \frac{e}{m_\pm}\left\{\left[-\frac{\partial V}{\partial r} + \frac{1}{c}(v_\vartheta H_\varphi - v_\varphi H_\vartheta)\right]\frac{\partial f_\pm}{\partial v_r} + \right.$$
$$+ \left[-\frac{\partial V}{r\partial\vartheta} + \frac{1}{c}(v_\varphi H_r - v_r H_\varphi)\right]\frac{\partial f_\pm}{\partial v_\vartheta} +$$
$$\left. + \left[-\frac{\partial V}{r\sin\vartheta\,\partial\varphi} + \frac{1}{c}(v_r H_\vartheta - v_\vartheta H_r)\right]\frac{\partial f_\pm}{\partial v_\varphi}\right\} = 0,$$
$$\Delta V = -4\pi e\int (f_+ - f_-)\,d\mathbf{v}, \quad \operatorname{rot}\mathbf{H} = \frac{4\pi e}{c}\int \mathbf{v}\,(f_+ - f_-)\,d\mathbf{v},$$

where the proper field of currents and the outer dipole magnetic field of the Earth are included in H.

The boundary conditions are:

$$V|_{r\to\infty} \to 0, \quad H_{r,\vartheta,\varphi}|_{r\to\infty} \to 0, \quad V|_{r\to a} \to \mathcal{E}/a, \quad \iint (f_+ - f_-)\,d\mathbf{r}\,d\mathbf{v} < \infty.$$

Here a is the radius and $\mathcal{E}$ is the electrical charge of the Earth. The integral condition requires the finiteness of the charge incident upon the circumterrestrial fully-ionized plasma.

We shall seek the stationary temperature distribution:

$$f = \rho\,(r, \vartheta)\,w\,(v_r^2)\,w\,(v_\vartheta^2)\,w\,(v_\varphi^2)\,F\,(r\sin\vartheta,\, v_\varphi),$$

where the function $\rho$, $\omega$ and F must be defined.

Limiting ourselves to problem symmetry, i.e. assuming $H_\varphi = 0$, and $\partial/\partial\varphi = 0$ we find

$$\rho = \rho_0 \exp(-eV/\theta), \quad w = (m/2\pi\theta)^{1/2}\exp(-mv^2/2\theta),$$
$$r\,\partial F/\partial r = (v_\varphi - reH_\vartheta/mc)\,\partial F/\partial v_\varphi, \quad \partial F/\partial\vartheta = (\operatorname{ctg}\vartheta\, v_\varphi + reH_r/mc)\,\partial F/\partial v_\varphi.$$

The solution of the equations for F is expressed by an arbitrary function, depending in a special way on the arguments

$$F = F\,[r \sin\vartheta\,(v_\varphi - (e/mc)\,A_\varphi)],$$

where $A_\varphi$ is the projection of the vectorial potential, provided the conditions $A_\varphi|_{r\to\infty} \to 0$, $A_\varphi \sin\vartheta|_\vartheta \to 0$. are utilized.

In order that the solution be thermal and contain the system's rotation, not only hard-bodied but its generalization too, it is necessary to concretize the function F as follows:

$$F = e^Q, \quad Q = (m\omega/2\theta)\,2r'v'_\varphi\,Ar - Av'^2_\varphi; \quad r' = r\sin\vartheta, \quad v'_\varphi = v_\varphi - \frac{e}{mc}A_\varphi,$$

where $\omega$, $\theta$ and A are constants, whose sense is determined from the following.

Let us represent the product $w\,(v'^2_\varphi)\,F$ in the form of a function with a velocity shift in space in the direction of parralels and with the "potential" function" S:

$$w\,(v'^2_\varphi)\,F\,(r\sin\vartheta\,v'_\varphi) = (m/2\pi\theta)^{1/2}\exp\{-(m/2\theta + \alpha)\,(v_\varphi - u_\varphi)^2 + S\};$$

we find

$$\alpha = A\,(r\sin\vartheta)^2, \quad u_\varphi = \frac{(m/2\theta)\,\omega r\sin\vartheta + A\,(r\sin\vartheta)^2\,(e/mc)\,A_\varphi}{m/2\theta + A\,(r\sin\vartheta)^2},$$

$$S = \frac{(m/2\theta)^2}{m/2\theta + A\,(r\sin\vartheta)^2}\left\{\omega^2r^2\sin^2\vartheta - 2\omega r\sin\vartheta\left(\frac{e}{mc}A_\varphi\right) - \right.$$
$$\left. - \frac{2\theta}{m}A\,(r\sin\vartheta)^2\,((e/mc)\,A_\varphi)^2\right\}.$$

We find the distribution of concentrations for both kinds of particles:

$$\rho_\pm(r,\vartheta) = \int f_\pm\,dv = \rho_0\,(1 + (2\theta/m)_\pm A_\pm r^2\sin^2\vartheta)^{-1/2}\exp\{-eV/\theta_\pm + S_\pm\}.$$

It is practical for the following to introduce the dimensionless quantities:

$$1/x = eM\omega/c\theta_-r, \quad \varphi = -eV/\theta_-, \quad \psi = -(e\omega/c\theta_-)\,A_\varphi r, \quad \mathcal{E}^* = \mathcal{E}/(M\omega/c)$$

Here M is the Earth's magnetic moment relative to which it is practical to express the quantities, $\mathcal{E}$ is the Earth's charge. Let us introduce also the following dimensionless coefficients: $\lambda = (4\pi e^2\rho_0/\theta_-)\,(eM\omega/c\theta_-)^2$,

$\varepsilon = (eM\omega^2/c^2\theta_-)^2$, $\alpha_\pm = (m/2\theta)_\pm\omega^2\,(eM\omega/c\theta_-)^2$, $\gamma_\pm = |(2\theta/m)\,A|_\pm\,(eM\omega/c\theta_-)^2$, $\beta_\pm = (A\theta/m^2)_\pm\,(\theta_-/\omega^2)$.

The knowledge of the stationary distribution function allows the formulating of the problem of electron and nucleus distribution by means of potentials $\varphi$ and $\psi$:

$$\Delta\varphi = \lambda\left[\frac{1}{\sqrt{\eta_+}}\exp\frac{\theta_-}{\theta_+}(\varphi+\Phi^+) - \frac{1}{\sqrt{\eta_-}}\exp(-\varphi+\Phi^-)\right],$$

$$\Phi^{\pm} = \pm\frac{\sin\vartheta}{\eta}\psi - \beta\frac{\sin^2\vartheta}{\eta}\psi^2 + \frac{\theta}{\theta_-}\alpha x^2\frac{\sin^2\vartheta}{\eta},$$

$$\mathrm{rot}_\varphi\,\mathrm{rot}\left(i_\varphi\frac{\psi}{x}\right) = \lambda\varepsilon\left[\frac{1}{\sqrt{\eta_+}}u_\varphi^+\exp\frac{\theta_-}{\theta_+}(\varphi+\Phi^+) - \frac{1}{\sqrt{\eta_-}}u_\varphi^-\exp(-\varphi+\Phi^-)\right],$$

where $\eta = 1+\gamma x^2\sin^2\vartheta$, $\alpha$, $\beta$, $\gamma$ и $\theta$ each take two values: + and —.

The unknown functions are subject to conditions

$$\varphi|_{x\to\infty}\to 0, \quad \psi|_{x\to\infty}\to 0, \quad \varphi|_{x\to\infty}\to \mathscr{E}/a,$$

$$\mathscr{E}_s = \frac{\lambda}{2}\int_0^\pi\int_0^\infty\left[\frac{1}{\sqrt{\eta_+}}\exp(\varphi+\Phi^+)\frac{\theta_-}{\theta_+} - \right.$$

$$\left. - \frac{1}{\sqrt{\eta_-}}\exp(-\varphi+\Phi^-)\right]x^2\,dx\sin\vartheta\,d\vartheta < \infty.$$

The convergence of the outstanding integral requires specific conditions of system's parameters.

Since at $x\to\infty$ $\varphi$ and $\psi$ approach zero, the integrand approaches the value $(1/\sqrt{\gamma_+})\exp(\alpha/\gamma)_+ - (1/\sqrt{\gamma_-})\exp(\alpha/\gamma)_-$. Since the ellipsoidal constant enters into $\gamma$, but does not in $\alpha$, it must be assumed that $\alpha$ and $\gamma$ are independent; then it follows from the convergence condition that $\alpha_+ = \alpha_-$ and $\gamma_+ = \gamma_-$ or $(\theta/m)_+ = (\theta/m)_-$, $A_+ = A_-$. Thus, the strong anisothermicity of the circumterrestrial plasma is the consequence of the natural boundary condition.

The problem posed admits the following simplifications:

1) a strong inequality takes place for all distances

$$\alpha x^2/(1+\gamma x^2) \ll 1,$$

which provides the possibility of neglecting the potential function of centrifugial forces;

2) The magnetic interaction of currents is a small quantity for the quantity characterizing this interaction is small:

$$\varepsilon = (eM\omega^2/c^2\theta_-)^2 \ll 1.$$

Thus we shall seek the solution in the form of a series $\psi = \sin\vartheta/x + \varepsilon\psi_1 + \ldots$,

where the first term reflects the magnetic dipole of the Earth. In the zero approximation relative to we have

$$\Delta\varphi = \lambda \frac{1}{\sqrt{\eta}} \left[ \exp \frac{\theta_-}{\theta_+} (\varphi + \Phi^+) - \exp (-\varphi + \Phi^-) \right],$$

$$\Phi^\pm = \pm \frac{\sin^2 \theta}{x\eta} - \beta_\pm \frac{\sin^4 \theta}{x^2 \eta}.$$

Inasmuch as $\rho_0$, entering into the parameter $\lambda$ — the density of particles at $x \to \infty$ (in interplanetary space) — depends on solar activity, a problem arises in regard to eigenvalues of the nonlinear equation for the potential. Let us examine two extreme cases.

CASE $\lambda < 1$. We shall seek the solution in the form of a series $\varphi = \varphi_0 + \lambda\varphi_1 + \lambda^2\varphi_2 + \ldots$, where $\varphi_0$ is the potential induced by charges beyond the considered region — in our case the aggregate charge of the Earth and of the atmosphere.

It is practical to introduce three distance zones

$$1 < x < 1/\sqrt{\lambda} < 1/\sqrt{\gamma} < \infty.$$

In the first zone we have in the zero and first approximations

$$\Delta\varphi_0 = 0, \quad \Delta\varphi_1 = \left[ \exp \frac{\theta_-}{\theta_+} (\varphi_0 + \Phi_0^+) - \exp (-\varphi_0 + \Phi_0^-) \right], \ldots,$$

$$\varphi_0 = \frac{\mathscr{E}^*}{x}, \quad \Phi_0^\pm = \pm \frac{\sin^2 \theta}{x} - \beta_\pm \frac{\sin^4 \theta}{x^2}.$$

The concentration of charged particles is expressed in the first approximation as:

$$\rho_\pm^{(1)} = \rho_0 \exp (\theta_-/\theta_+) (\mp \mathscr{E}^*/x \pm \sin^2 \theta/x - \beta_\pm \sin^4 \theta/x^2).$$

These functions have a maximum, whose position is determined by the formulas:

$$x_+ = 2\beta_+ \sin^4 \theta/(-\mathscr{E}^* + \sin^2 \theta), \quad x_- = 2\beta_- \sin^4 \theta/(\mathscr{E}^* - \sin^2 \theta).$$

If the aggregate charge of the Earth and of the atmosphere is positive, but relatively small ($\mathscr{E}^* < 1$), there exist maxima of nucleus and electron concentration depending upon a soecific form of distance (see Fig.1 next page).

In the second and the third distance zones a screening effect of the electrostatic potential occurs with a different screening law.

CASE $\lambda > 1$. Let us take advantage of the expansion $\varphi = \varphi_0 + \varphi_1/\lambda + \varphi_2/\lambda^2 + \ldots$. From the initial equation for $\varphi$ we find for the zero approximation

$$\varphi_0 = -\frac{\sin^2\vartheta}{x\eta} - \frac{\beta_- - (\theta_-/\theta_+)\beta_+}{1 + \theta_-/\theta_+}\frac{\sin^4\vartheta}{x^2\eta},$$

$$\eta = 1 + \gamma x^2 \sin^2\vartheta$$

and for the first approximation

$$\varphi_1 = \sqrt{\eta}\left(1 + \frac{\theta_-}{\theta_+}\right)^{-1} \Delta\varphi_0 \exp\left(\frac{(\theta_-/\theta_+)(\beta_- + \beta_+)}{1 + \theta_-/\theta_+}\frac{\sin^4\vartheta}{x^2\eta}\right).$$

Determining hence the concentrations of charged particles, we conclude that in the zero approximation

$$\rho_+^{(0)} = \rho_0 \exp\left\{-\frac{(\theta_-/\theta_+)(\beta_- + \beta_+)}{1 + \theta_-/\theta_+}\frac{\sin^4\vartheta}{x^2\eta}\right\} = \rho_-^{(0)}$$

they are equal and approach a constant value as x rises. In the following first approximation we find for the relative concentration

$$(\rho_+^{(1)} - \rho_-^{(1)}) / \rho_0 = \Delta\varphi_0/\lambda,$$

where $\varphi_0$ is the zero approximation.

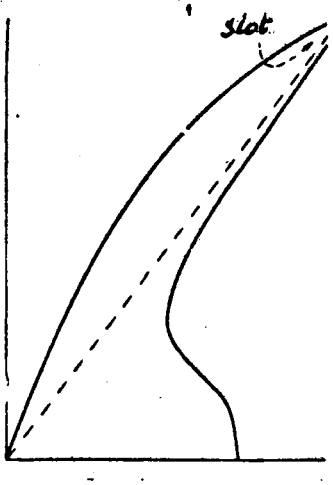


Fig. 1. Line of maxima for the concentration of nuclei and electrons (case $\lambda < 1$, $0 < \vartheta^* < 1$)

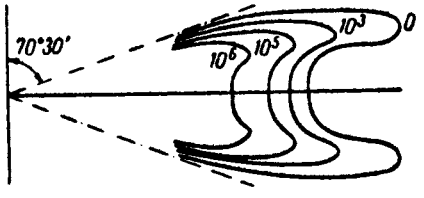


Fig. 2.- Lines of equal concentrations of the quantity $(\rho_+^1 - \rho_-^1)/\rho_0$ (case $\lambda > 1$).

The right-hand part, as a function of $\vartheta$ and x is analyzed graphically.

Identifying the boundary of the end of positive charges ($\Delta\varphi_0 = 0$) with that of the first radiation belt from the side of great distances, we may determine the value of the ellipsoidal constant A. From the conditions $\Delta\varphi_0 = 0, \vartheta = \pi/2$, and assuming that for the indicated boundary and by order of magnitude $r \sim 1 \cdot 10^9$ cm, $\theta_- \sim 10^6$ ev, we find $A \sim 1.3 \cdot 10^{-40}$ sec$^2$/cm$^4$.

It follows from the graphical analysis that:

1.- The presence of a belt with predomination of nuclei over electrons (see Fig. 2) is a fact. The quantity $(\rho_+^{(1)} - \rho_-^{(1)})/\rho_0$ reaches in it values of the order of $10^6$. The angle $\vartheta$, from which it begins, is equal to 70° 30'.

2.- Electrons predominate beyond this belt, for here $\Delta\varphi_0 < 0$. There is within approximately the same aperture angle $\delta$ an outlined maximum for electrons immediately beyond the boundary of the first belt.

The solution obtained satisfies the neutrality condition of the whole system (Earth, atmosphere and the fully-ionized circumterrestrial plasma).

In the course of the total preceding examination it was assumed that the rotation and magnetic moment axes coincide. If $\delta\vartheta$ denotes the angle between these directions, the function $\psi$, defining the vectorial potential of the Earth's dipole, is expressed by the angle

$$\delta\theta: \quad \psi = \sin(\vartheta + \delta\theta)/x.$$

That is why substitution of $\sin^3\vartheta \to \sin\vartheta \sin(\vartheta + \delta\vartheta)$. must be made in all the calculations.

**** THE END ****

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in the name of
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